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FOR THE

NOBEL PRIZE IN PHYSICS FOR I992

submitted by

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PART I:

SANTILLI'S LIE-ISOTOPIC AND LIE-ADMISSIBLE GENERALIZATIONS OF GALILEI'S RELATIVITY FOR CLASSICAL DYNAMICAL SYSTEMS WITHIN PHYSICAL MEDIA

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THE NOMINATION

Physics is a discipline that will never admit *final theories* . No matter how authoritative current theories are, their generalization is only a matter of time.

Physics is also a discipline centrally dependent on mathematical elaborations of the physical reality in a quantitative form suitable for experimental verification.

The contemporary relativities, *Galilei's, Einstein's special and Einstein's general relativities*, and related physical theories, are based on an articulated body of mathematical methods comprising:

I) *Algebras* , e.g., Lie's theory in its various branches such as enveloping algebras, Lie algebras, Lie groups, representation theory, etc.;

II) *Geometries* , e.g., the Euclidean, symplectic, and Riemannian geometries; and

III) *Mechanics* , e.g., conventional nonrelativistic and relativistic Lagrangian and Hamiltonian mechanics;

and others.

Professor **Ruggero Maria Santilli**, while working first at the *Istituto di Fisica Teorica* of the *Universitá degli Studi* , Torino, Italy (where he obtained his PhD in Physics), then at the *Department of Mathematics* of *Harvard University* , Cambridge, MA, USA, and more recently at the *International Centre for Theoretical Physics* of Trieste, Italy, has achieved an unprecedented series of discoveries consisting, first, of the identification of new mathematical methods, including:

I') *Certain generalizations of Lie algebras called of isotopic type* ;

II') *Isotopic generalizations of the Euclidean, symplectic and Riemannian geometries*; and

III') *Isotopic generalizations of conventional nonrelativistic and relativistic, classical and quantum mechanics* .

Then, via the use of these broader mathematical tools, Santilli succeeded in constructing certain *generalizations-coverings of Galilei's relativity, Einstein's special relativity, and Einstein's general theory of relativity* for novel physical conditions in which the conventional relativities are inapplicable.

As well known, the conventional relativities describe particles which can be approximated as being point-like while moving in vacuum under action-at-a-distance, potential forces (historically called *exterior dynamical problem* , see Sect. I).

Santilli's new relativities describe instead the most general known physical systems, namely, extended and therefore deformable particles while moving within generally inhomogeneous and anisotropic physical media, resulting in equations of motion that are nonlinear, nonlocal, as well as not representable via the usual Lagrangian or Hamiltonian (historically referred to as *interior dynamical problem*, see Sect. I).

Also, ***Santilli's new relativities are a covering of the conventional ones*** in the sense that: 1) they are based on more general mathematical methods; 2) they represent structurally more general physical conditions; and 3) they admit the conventional relativities as particular cases.

There is little doubt that ***Santilli's discoveries are among the most important ones which can be brought to the attention of the NOBEL COMMITTEE***. In actuality, it appears that Santilli's discoveries are unprecedented in physics as an achievement by one single individual. In fact, virtually all discoveries made by physicists until now were based on mathematical tools previously established by mathematicians. The unprecedented aspect of Santilli's discoveries is that, before being in a position to generalize conventional relativities, he had to discover all needed new mathematical methods because unavailable in the mathematical literature for the needed application: the treatment of nonlinear, nonlocal-integral, nonlagrangian and nonhamiltonian systems of the interior dynamical problem.

The purpose of this presentation, specifically written for the NOBEL COMMITTEE, is multifold. First, we would like to indicate the fundamental novelty, comprehensive character, and historical dimension of the discoveries.

Second, Santilli has written in the topic some seven monographs and over 100 articles in numerous international Journals. By adding the contributions of other independent scientists, we are dealing with a field that has surpassed the mark of ten thousand pages of published research. The second objective of this presentation is, therefore, that of identifying for the NOBEL COMMITTEE the most salient aspects of the discoveries and their original reference among such a disparate literature.

Third, the novel scientific edifice emerging from Santilli's discoveries implies a generalization of the entirety of contemporary physics, including generalizations of: classical nonrelativistic and relativistic mechanics, nonrelativistic and relativistic quantum mechanics, quantum field theory, gravitation, classical and quantum statistics, etc. It is easily predictable that in a scientific scene of this dimension, we have a spectrum of conditions, including: discoveries which can be safely considered as established at this writing; discoveries in need of additional theoretical and experimental studies; and others only at their initiation.

> ***This Nomination of Professor RUGGERO MARIA SANTILLI for the NOBEL PRIZE IN PHYSICS FOR 1992 is solely based on those discoveries which are fully established at this writing on both grounds of mathematical consistency and physical validity, and which, as outlined in this Part I, consists of Santilli's classical generalization of Galilei's relativity for nonlinear, nonlocal and nonhamiltonian dynamical systems of the interior problem.***

according to the following primary publications:

1) The discovery originally appeared in the memoir (Santilli (1978a); see Document A for its front page); and was then subjected to a variety of specialized studies in a number of papers identified in this Part I;

2) The discovery was then presented in all the necessary details in the four monographs:

R. M. Santilli, ***Foundations of Theoretical Mechanics,***

Volume I: ***The Inverse Problem in Newtonian Mechanics (1978b)*** ,

Volume II: ***Birkhoffian Generalization of Hamiltonian Mechanics (1982a)***, and

printed by *Springer-Verlag* of Heidelberg, Germany,

R. M. Santilli, ***Lie-admissible Approach to the Hadronic Structure***

Volume I: ***Nonapplicability of Galilei's and Einstein's Relativities ? (1978c)***,

Volume II: ***Generalizations of Galilei's and Einstein's Relativities ? 1981a)***

published by *Hadronic Press Inc.*, Palm Harbor, FL 34682-1577 USA, and

3) The discovery was then finalized in the memoir (Santilli (1988a)). Its novel mathematical structures were studied in detail, first, in the memoir (Santilli (1988b)), and then in the two memoirs appeared in a mathematical Journal (Santilli (1991a, b). The discovery was then finalized in its physical contents in the two additional monographs:

R.M.Santilli, ***Isotopic Generalizations of Galilei's and Einstein's Relativities***

Volume I: ***Mathematical Foundations (1991c)***

Volume II: ***Classical Isotopies (1991d)***

In our opinion, the content of the two monographs published by Springer-Verlag in 1978 and 1982 is sufficient to warrant, alone, a NOBEL PRIZE IN PHYSICS. In fact, the title of Volume II reads ***"Birkhoffian Generalization of Hamiltonian Mechanics"*** and does indeed present a completely new physical discipline (see Section IV for a brief outline). Similarly, the title of Chapter 6, p. 199, Volume II, reads ***"Generalization of Galilei's Relativity"*** and presents a generalization that appears in print for the first time after about four centuries from the original Galilean conception.

As a result of this occurrence, ***copies of Prof. Santilli's two monographs published by Springer-Verlag are enclosed as an integral part of this Nomination*** . The more recent monographs 3) will be separately mailed to the NOBEL COMMITTEE as addenda.

Part II, which is under preparation for the NOBEL COMMITTEE, outlines *Santilli's isotopic generalization of Einstein's special relativity for light and/or relativistic systems of extended-deformable particles moving within inhomogeneous and anisotropic physical media* , and it is scheduled for submission sometime in 1992. This latter new relativity is mathematically consistent but, unlike the Galilean case, its novel predictions (e.g., that for a redshift of light propagating within inhomogeneous and anisotropic transparent media, and others) needs specific experimnental verifications.

Part III, also in preparation for the NOBEL COMMITTEE for delivery sometime in 1992, outlines *Santilli's isotopic generalization of Einstein's gravitation for the most general known nonlinear, nonlocal and nonlagrangian interior gravitational conditions, as expected, say, for a star undergoing gravitational collapse or for any interior gravitation at large.* This third new relativity is also mathematically consistent at this writing but its physical consistency, in addition to the experimental verifications for the local relativistic interior behavior indicated above, requires additional studies connected to the numerous and now vexing problematic aspects of Einstein's gravitation.

Part IV, also in preparation for the NOBEL COMMITTEE, outlines *the operator formulation of Santilli's coverings of Galilei's and Einstein's special relativities for elementary particles with extended wavepackets when in conditions of total mutual penetration, as conceivable for the hadronic structure, which result in expected, short range, nonlinear, nonlocal and nonhamiltonian internal effects without any visible effect in the exterior dynamics* . These studies, which have resulted in a generalization of quantum mechanics called *hadronic mechanics* , are also mathematically consistent at this writing, but in need of a number of additional theoretical elaborations and experimental verifications.

A final Part V may be prepared for the NOBEL COMMITTEE at some future time on certain ongoing efforts to achieve an isotopic generalization of unified gauge theories for the possible inclusion of gravitational and strong interactions, known under the name of *iso-grand-unification* .

This Nomination for the NOBEL PRIZE IN PHYSICS OF 1992 is solely referred to this Part I. Nevertheless, the NOBEL COMMITTEE should be aware of the additional discoveries outlined in the remaining parts, because important to reach a mature judgment on their dimension, depth, implications and interrelations.

It may be appropriate here to recall that Santilli's discoveries were called

"Truly epoc making", by Prof. H.P.Leipholz, Univ. of Waterloo, Canada
(official reviewer for Springer-Verlag, see enclosed Document B).

In recognition of his discoveries, Santilli received ***two Gold Medals***, one from the City of Orléans, France, and another from the city of Campobasso, Italy, in conjunctions with international Conferences in which he presented his discoveries (see Document C).

But the biggest honor was granted until now by the Estonian Academy of Science in Tartu which, in the occasion of an International Conference in algebras of 1989, prepared two official charts on the most historical contributions in physics and mathematics from 1800 till today (Document D) which were presented at the Conference during the opening talk of the organizers and subsequently printed in 1990 (ISSN0134-627X). As the NOBEL COMMITTEE can see, the name RUGGERO SANTILLI is listed with the year 1967 of initiation of the discoveries that lead to this Nomination, jointly with the best names in the history of physics and mathematics, such as GAUSS (1820), CAUCHY (1847), HAMILTON (1843), CAYLEY (1854), LIE (1880), POINCARE' (1884), CARTAN (1894), NOETHER (1929), EDDINGTON (1928), WEYL (1926), DIRAC (1928), JORDAN (1932), VON NEUMANN (1934), WIGNER (1934), ALBERT (1948), and others.

SECTION I: INTRODUCTION.

I.1: THE FIELD OF UNEQUIVOCAL VALIDITY OF CONVENTIONAL RELATIVITIES.

Galilei's relativity (see Galilei (1638), Newton (1687) and, for a contemporary account, Sudarshan and Mukunda (1974)), *Einstein's special relativity* (see Lorentz (1904), Poincaré (1905), Einstein (1905), Minkowski (1913) and, for a historical review, Pauli (1921)) and *Einstein's general relativity* (see Riemann (1868), Einstein (1916) and, for a historical account, Pauli (loc. cit.)) were conceived for physical conditions referred to by Lagrange (1788), Hamilton (1834), Jacobi (1837) and other Founders of contemponary physics, as those of the *exterior dynamical problem*, that is, the study of particles which can be well approximated as being point-like, while moving within the (homogeneous and isotropic) vacuum, under action-at-a-distance interactions derivable from a potential energy.

The point-like character of the particles implies the exact validity of conventional local-differential geometries, such as the symplectic, affine or Riemannian geometries. The action-at-a-distance, potential nature of the interactions then implies the exact validity of all conventional Lagrangian-Hamiltonian disciplines, such as the conventional nonrelativistic and relativistic, discrete or continuous, classical or quantum mechanics.

An overwhelming amount of experimental evidence has nowadays established the validity of the conventional relativities in the arena considered beyond any possible doubt. It is here appropriate to recall, as a classical illustration, the majestic successes of the NASA missions throughout our Solar system and, as a quantum mechanical illustration, the equally majestic successes in the description of the atomic structure.

The exact validity of conventional relativities, within the above identified conditions, is assumed by Santilli's as the sound foundations of his research. The NOBEL COMMITTEE should therefore expect *no conflict whatever* between conventional relativities and Santilli's generalizations, but only a continuity of mathematical and physical thought, as we shall see.

I.2: THE DIFFERENT FIELD OF VALIDITY OF SANTILLI'S COVERING RELATIVITIES.

Santilli has devoted his research life to the study of physical conditions fundamentally different and substantially more complex than the above. In particular, he has studied conditions which were historically referred to by Lagrange (loc. cit.), Hamilton (loc. cit.), Jacobi (loc. cit.) and other Founders, as those of the *interior dynamical problem*, that is, the study of extended particles which cannot be approximated as being point-like, while moving within generally *inhomogeneous* and *anisotropic* physical media under action-at-a-distance, potential forces, as well the additional contact forces with the physical media for which the notion of potential has no meaning.

Lagrange and Hamilton knew well that the contact forces between extended objects and the physical media in which they move are outside the representational capabilities of their functions and, for this reason, they formulated their historical equations with external terms. In fact, for a system of N particles represented by the index a = 1, 2, ..., N, in three-dimensional Euclidean space with local coordinates $r = (r_{ka})$, k = 1, 2, 3 (= x, y, z)[1], the historical Lagrangian and Hamilton's equations *are not* those given in the contemporary texbooks of physics and mathematics, but instead by the forms

$$\frac{d}{dt}\frac{\partial L(t. r, \dot{r})}{\partial \dot{r}_{ka}} - \frac{\partial L(t, r, \dot{r})}{\partial r_{ka}} = F_{ka}(t, r, \ddot{r}, ...), \tag{I.1a}$$

$$\dot{r}_{ka} = \frac{\partial H(t, r, p)}{\partial p_{ka}}, \qquad \dot{p}_{ka} = -\frac{\partial H(t, r, p)}{\partial r_{ka}} + F_{ka}(t, r, \dot{r}, ...) \tag{I.1b}$$

$$k = 1, 2, 3 \ (= x, y, z), \qquad a = 1, 2, ..., N,$$

where the external terms F_{ka} represent precisely the contact, nonlagrangian/nonhamiltonian forces of our physical reality. Similarly, Jacobi formulated his historical theorem, not for the "contemporary Lagrange's and Hamilton's equations", those *without* external terms, but for the original ones *with* external terms.

With the passing of time, the external terms were removed as a result of a historical process still ignored by historians until now, such as: the advent of Lie's theory (1983), the classical and quantum mechanical successes for the description of exterior planetary and atomic systems, respectively, and other reasons. In this way, Lagrange's and Hamilton's equation acquired the contemporary "truncated form" without external terms.

As a result of this process, the original, historical distinction between the exterior and interior dynamical problem was progressively lost, up to the contemporary scientific scene which is virtually without any remnant of the historical distinction.

Santilli essentially dedicated his research life to a comprehensive classical and quantum mechanical study of historical equations (I.1) *with* external terms. In fact, he first identified their algebraic character as being that of a generalization of Lie algebra called *Lie-admissible algebras* (Sect. III.3) and discovered their underlying new geometry, which he called *symplectic-admissible geometry* (Appendix A),. Santilli then succeeded in identifying their operator image (see the forthcoming Part IV). This first group of methods, now known as *Santilli's Lie-admissible formulations* (Sect. III.3 and Appendix A), is particularly suited for the direct study of Lagrange's and Hamilton's interior problem in its original conception, that is, under open-nonconservative conditions.

Santilli then identified a second group of methods, now known as *Santilli's Lie-*

[1] For simplicity, we shall ignore any disctinction between covariant and contravariant indeces for coordinates $r = (r_{ka})$ and momenta $p = (p_{ka})$, but introduce the dictinction later on for the unified notation in phase space $a = (a^{\mu}) = (r, p)$, $\mu = 1, 2, ..., 6N$

isotopic formulations, which essentially consist of an alternative approach in which the external terms are removed, and replaced by a generalized unit of the theory, by resulting in a structural generalization of Lie algebras, symplectic geometry and Hamiltonian mechanics, called by Santilli *Lie-isotopic algebras, symplectic-isotopic geometry and Birkhoff mechanics*, as outlined in Sect. IV. Santilli also succeeded in identifying the operator countepart of these alternative formulations, which is outlined in the forthcoming Part IV. These latter formulations are particularly suited for "closing" Eq.s (I.1) via the addition of the external media, thus resulting in isolated systems verifying all conventional total conservation laws, while the internal forces are nonlinear, nonlocal and nonhamiltonian (Sect. II).

The technical foundations of all these studies are provided by the so-called *conditions of variational selfadjointness*, which are presented in details in the enclosed first monograph by Santilli under the title ***Foundations of Theoretical Mechanics, Vol. I: The Inverse Problem in Newtonian Mechanics***, published by *Springer-Verlag*, Heidelberg (1978b). As the NOBEL COMMITTEE can see, this is a very scholarly work providing the first comprehensive presentation of the *necessary and sufficient conditions for given forces to admit a potential or, more generally, for given equations of motion to admit a Lagrangian or a Hamiltonian*.

The NOBEL COMMITTEE should also be aware of the historical search conducted by Santilli in the scientific libraries of Cambridge, Massachusetts as an essential part of this monograph. In fact, the paternity of the integrability conditions for the existence of a Lagrangian were essentially unknown in the 70's, with contrasting quotations generally existing in advanced mathematical papers. In his comprehensive library search, which lasted from 1975 to 1978, Santilli succeeded in establishing that Helmholtz (1887) had been the originator of the conditions of variational selfadjointness, and then identified all subsequent contributions (see Vol. I of the enclosed monographs, pages 12, 13).

The conditions of variational selfadjointness are the true technical foundations for both the Lie-admissible and the Lie-isotopic formulations, inasmuch as they provide all the necessary quantitative means for studying the structure of any given force, the conditions when it is reducible to the Lie-isotopic formulations, and the conditions under which the more general Lie-admissible methods are requested.

The NOBEL COMMITTEE can find in Document E the outline of a post-graduate course Santilli taught in 1978 in the field at the *Lyman Laboratory of Physics* of *Harvard University*.

I.3: INEQUIVALENCE OF THE INTERIOR AND EXTERIOR PROBLEMS.

One of the first introductory points the NOBEL COMMITTEE can find in Santilli's writings is the proof of the inequivalence of the interior and exterior problems (Santilli (1978a, c), (1982a), (1985c), (1988a), (1991c)).

In fact, the exterior problem is based on the point-like abstraction of particles under interactions derivable from a potential V and are representable in their first-order form via the familiar Hamiltonian vector-fields

$$a = (\dot{a}^{\mu}) = \begin{pmatrix} \dot{r}_{ka} \\ \\ \dot{p}_{ka} \end{pmatrix} = \Phi = (\Phi^{\mu}(a)) = \begin{pmatrix} p_{ka}/m_a \\ \\ -\dfrac{\partial V}{\partial r_{ka}} \end{pmatrix} \qquad \text{(I.2)}$$

$$a = (a^{\mu}) = (r, p), (r_{ka}, p_{ka}), \quad \mu = 1, 2, ..., 3N, \quad k = 1, 2, 3, (= x, y, z)$$

and, as such, it implies the exact validity of conventional local-differential geometries, such as the symplectic, affine and Riemannian geometries.

On the contrary, interior dynamical problems describe, say, a satellite during re-entry in Earth's atmosphere, where, in addition to the conventional local-differential forces derivable from a potential V, experimental evidence establishes the existence of the contact interaction with the medium which can be reduced to local-differential nonlinear and nonhamiltonian forces $F_k(t, r, p,...)$, plus additional nonlocal-integral forces also evidently not derivable from a Hamiltonian (see, e.g., Hofstadter et al. (1970), Fijimura et al. (1971), and quoted papers)

$$a = (\dot{a}^{\mu}) = \begin{pmatrix} \dot{r}_{ka} \\ \\ \dot{p}_{ka} \end{pmatrix} = \Gamma = (\Gamma^{\mu}(t, a, ..)) = \begin{pmatrix} p_{ka}/m_a \\ \\ -\dfrac{\partial V}{\partial r} + F_{ka}(t, r, p, ...) + \int_{\sigma} d\sigma \mathcal{F}_{ka}(t, r, p, ...) \end{pmatrix} \qquad \text{(I.3)}$$

Thus, interior dynamical systems are characterized by the most general known systems of differential equations which are: 1) nonlinear and nonlocal in all variables; 2) nonlagrangian and nonhamiltonian, in the sense that the conventional functions $L(t, r, \dot{r})$ or $H(t, r, p)$ are insufficient to represent the system; and 3) nonnewtonian in the sense that the acting forces generally dependent on the accelerations.

As the NOBEL COMMITTEE can see, systems (I.3) can be readily represented by the original equations (I.1), where L or H represent the kinetic energy T as well as the potential energy V, L = T - V or H = T + V, while the external terms F_{ka} represent all the nonpotential forces.

The inequivalence of the interior and exterior problems is then established at all levels of studies. In fact:

A) *Topologically*, the nonlocal character of systems (3) imply the inapplicability of all basic geometries of contemporary physics, the symplectic, affine and Riemannian geometries because of their strict local-differential nature;

B) *Algebraically*, the nonlinear, nonlocal and nonhamiltonian character of the systems imply the inapplicability of the conventional canonical formulation of Lie's theory;

C) *Analytically*, the nonlagrangian and nonhamiltonian character implies the inapplicability of current analytic mechanics;

etc.

Even by approximating the nonlocal-integral forces with local-differential expressions (which are usually done via power series expansions in the velocities truncated to a sufficiently high power), the nonlagrangian and nonhamiltonian character of the systems persists. In fact, it is well known in engineering circles (but not yet in physical circles) that the computerized guidance systems of missiles in atmophere may require contact forces *up to the tenth power in the velocity and more* , thus being strictly nonlagrangian and nonhamiltonian.

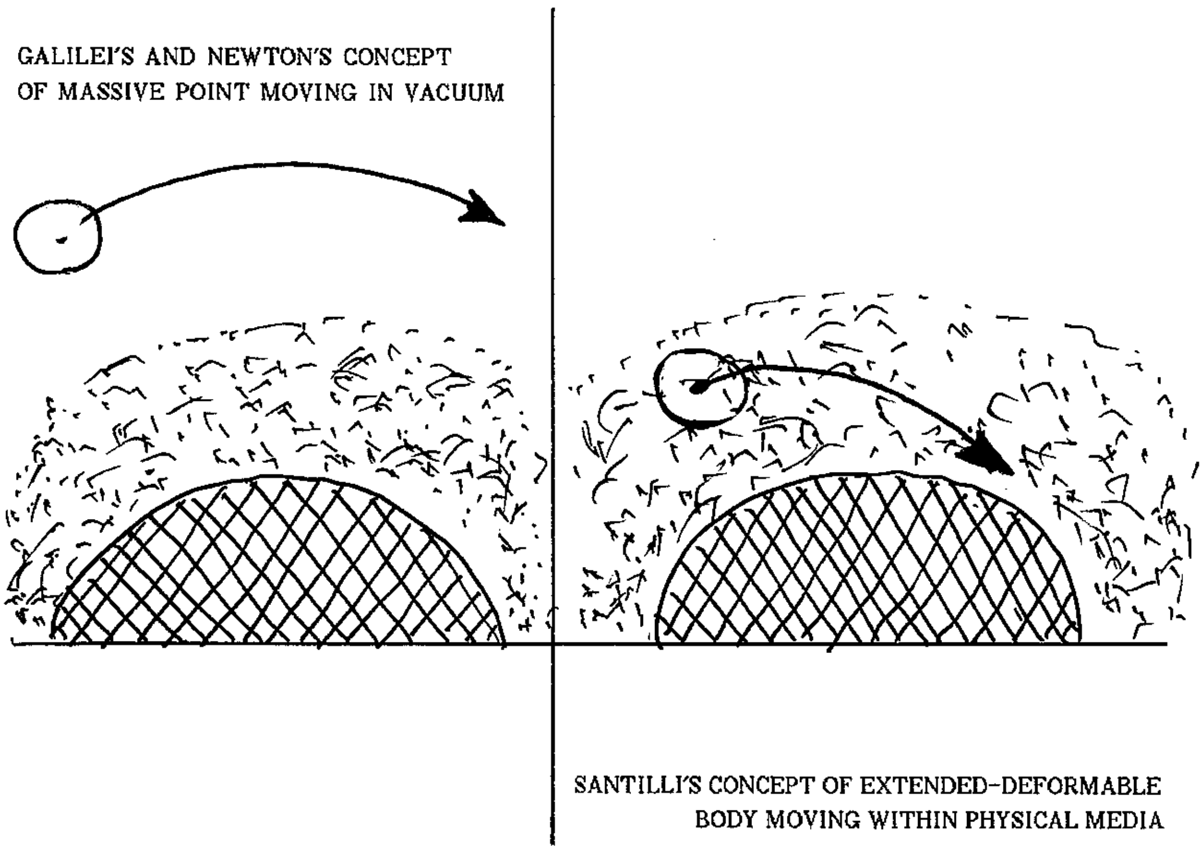

FIGURE I.1: A schematic view of the funndamental concepts in conventional and Santilli's relativities. Consider an object moving in empty space, such as a satellite in a stationary orbit around our Earth. Since motion occurs in vacuum, the extended character of the object and its actual shape do not affect its dynamical evolution. One recovers in this way ***the historical notion of "massive point" by Galilei (1638) and Newton (1687)***, namely, the satellite can be assumed to be a massive point concentrated in its center of gravity without any consequential approximation in the dynamics. This centuries hold concept has profound, contemporary, topological implications. In fact, it implies the exact validity of the local-differential geometries of contemporary mathematics, such as the differential, affine and Riemannian geometries. The Newtonian equations of motion are then given by Eq.s (I.2). The exact validity of Galilei's relativity, Einstein's special relativity and Einstein's gravitation for the satellite in exterior conditions is then consequential.

Consider now the satellite during re-entry in Earth's atmosphere. The dynamical conditions are then profoundly altered. In fact, the actual shape of the satellite now directly affects its dynamical evolution (e.g., spherical and nonspherical satellites of equal mass have essentially different trajectories in atmosphere). As a result, the extended character of the satellite must be represented in the equations of motion. Moreover, perfectly rigid objects do not exist in Nature. One therefore has deformations of the shape of the satellite which must also be taken into consideration. We reach in this way ***Santilli's concept of extended and therefore deformable object moving within generally inhomogeneous and anisotropic physical media*** of equally historical character which is at the foundation of this Nomination. The mathematical implications of the latter concept are far reaching. In fact, the representation of the shape in the dynamical evolution requires forces of integral type as in Eq.s (I.3) where σ now represents the surface of the satellite. In turn, this implies the irreconcilable inapplicability of conventional geometries, such as the symplectic, affine and Riemannian geometries, because of their strictly local-differential topology. The inapplicability of conventional relativities is then consequential, as outlined in the text.

While a member of the *Department of Mathematics* of *Harvard University* in the late 70's, Prof. Santilli studied all available efforts by pure mathematicians in the construction of the so-called "integral topologies" and "integral geometries" for the purpose of ascertaining their effectiveness in the treatment of systems (I.3). He found none available in the pure mathematical literature which would verify all his requirements, including the conditions of: a) admitting nonlocal-integral forces of nonlagrangian and nonhamiltonian character; b) being simple in use and effective in physical applications; and, last but not least, c) permitting the constructions of covering relativities. He therefore constructed novel geometries and mathematical tools for the needed quantitative treatment of *"extended"* and *"deformable"* objects moving within *"inhomogeneous"* and *"anisotropic"* physical media (see Sect.s III and IV). The construction of generalized relativities was then consequential (Sect. V).

These experimental facts establish that interior trajectories are structurally beyond the representational capabilities of the symplectic and Riemannian geometries (a property also known as the *Cartan's legacy*) , thus establishing the need for suitable generalizations.

1.4: IRREDUCIBILITY OF THE INTERIOR TO THE EXTERIOR DYNAMICAL PROBLEM.

When exposed to interior systems (I.3), contemporary physicists generally provide all conceivable efforts in reducing them to the simpler form (I.2). Santilli (1985c) has proved that such a reduction is inconsistent and not realizable in technical terms.

First, when exposed to systems of type (I.3), physicists tend to perform their transformation, from the original coordinate system r of their experimental detection, into an imaginary frame r' in which the systems become Hamiltonian. Under a number of approximations and conditions (locality, regularity and analyticity in a star-shaped region), the existence of such a transformation is ensured by the *Lie-Koening Theorem* (Santilli (1982a)). However, it is a mere mathematical curiosity, because the original system is nonhamiltonian as well as nonlinear. Thus, the transformation $r \Rightarrow r'$ must necessarily be *noncanonical* , as well as highly *nonlinear* . This renders inapplicable all conventional relativities to the hypothetical, transformed system, evidently because the transformed frame r′ is highly noninertial, as well as nonrealizable in experiments (e.g., $r' = a \exp[b \sinh (c\, r)]$, $a, b, c \in \Re$).

Santilli (loc. cit.) therefore insists that ***systems (I.3) must be represented in the physical coordinates r of their experimental detection*** (which he calls "direct representation"). Besides, systems (I.3) are nonlocal-integral, in which case the Lie-Koening Theorem is known to be *inapplicable* and the reduction to a Hamiltonian form impossible.

After recognizing the impossibility of effectively reducing systems (I.3) to the simpler form (I.2) treatable via current relativities, contemporary physicists claim that their differences are *"illusory"* (sic) because, when a macroscopic body of the interior dynamical problem, such as a satellite during re-entry, is reduced to its elementary particle constituents, one recovers point-like particles in stable orbits under potential interactions, with the consequential validity of conventional geometries, disciplines and relativities.

In an invited talk at an International Conference held in Calcutta in 1985, Santilli (1985c) presented a series of *"No Reduction Theorems"* which establish the impossibility of any consistent reduction of a classical, nonconservative and nonlagrangian-nonhamiltonian system to a collection of conservative, Lagrangian-Hamiltonian particles. Viceversa, he proved that a (finite) collection of elementary particles in stable orbits and in unitary time evolution simply cannot reproduce a mascroscopic system which is in highly nonconservative conditions and not representable by a Hamiltonian.

When the impossibility of a consistent elimination of the interior nonlocality is finally acknowledged, contemporary physicists still attempt other mechanisms in the hope of salvaging established doctrines for interior conditions.

One of them is the addition of an "integral potential" to conventional Lagrangians and Hamiltonians. The simplistic argument is that the salvaging of the canonical formalism implies the preservation of conventional relativity. Santilli (1978c) has proved the inconsistency of these latter attempts on numerous mathematical and physical grounds reviewed in Sect. II.2 (see Footnote[2] in particular), such as the invalidation of the conventional local-differential topology with consequential loss of topological symmetries, as well as the necessary impact on the *exterior* trajectory caused by the *internal* effects (because of its "potential" interpretation), which is against clear physical evidence.

The mathematical roots of Santilli's "No Reduction Theorems" is the evidence that the unstable orbit, say, of a satellite during re-entry with monotonically decaying angular momentum, simply cannot be decomposed into a collection of stable orbits, each one with conserved angular momentum. Viceversa, a collection of stable orbits each with conserved angular momentum simply cannot reproduce a macroscopic body with monotonically decaying angular momentum.

The physical roots are given by the ***legacy of Fermi (1949), Bogoliubov (1960), and other Founders of particle physics on the ultimate non-locality of the structure of strongly interacting particles***. In fact, while the atomic constituents are at large mutual distances when compared to their wavelength, the hadronic constituents must necessarily be in conditions of total mutual penetration and overlapping because their wavelength is precisely of the order of magnitude of the size of all hadrons (about $1\ F = 10^{-13}$ cm). Thus the atomic structure is a typical example of *exterior quantum mechanical problems*, while the hadronic structure is expected to be a typical case of *interior quantum mechanical problems*.

As well known, current theories on the structure of hadrons are dominated by the hypothesis that the constituents of hadrons are the *quarks* (see the reprints of the original contributions edited by Lichtenberg et al. (1980)). Now, even though there is experimental evidence (Bloom et al (1969)) that the hadronic constituents have a point-like charge structure (for which NOBEL PRIZES were recently granted), "point-like wave packets" do not exist in Nature. Quarks, to be physical particles, must therefore have extended wavepackets with the dimension of the entire hadron. The historical legacy on the ultimate nonlocality of strong interactions then follows.

One may argue from the clear successes of the quark theories that such nonlocal effects could be small. Nevertheless, if one passes to more limiting conditions, they simply are not ignorable. The NOBEL COMMITTEE can consider in this respect the core of a collapsing star, in which we have not only total mutual penetration of the wavepackets of the particles constituents, but also their compression in very large numbers in an extremely small region of space. Under these conditions, the validity of the historical legacy on the ultimate nonlocality of the structure of matter becomes beyond any credible scientific doubt.

This illustrates the ***necessity of studying the interior dynamical problem at all its levels, nonrelativistic, relativistic, gravitational, classical and quantum mechanically***, precisely as done by Santilli.

Moreover, despite their successes, quark theories are still afflicted by fundamental, now vexing, open problems. As an example, all nonrelativistic quark theories have a finite nonnull probability of tunnel effects for free quarks when near the infinite potential barrier (Chattarjee et al. (1986)), which is contrary to experimental evidence. This occurrence is a necessary consequence of the assumption of quantum mechanics in general, and Heisennberg's uncertainty principle in particular, in the interior and in the exterior problems of hadrons. The same nonnull probability of tunnel effects is expected to persist at all subsequent levels of treatment, such as that of QCD, because inherent in Heisenberg's uncertainty principle.

Explicitly stated, Heisenberg's uncertainty principle implies the consequence, beyond any reasonable doubt, that ***current quark theories have a finite nonnull probability that the ordinary protons and neutrons should spontaneously emit free quarks, which is evidently contrary to experimental evidence***.

In Part IV of this Nomination we shall review for the NOBEL COMMITTEE the fact that this (and other) vexing open problems of current quark theories may be due precisely to their lack of treatment of the historical legacy on the ultimate nonlocality of the hadronic structure. In fact, short range, nonlocal effects can be admitted only in the interior problem, and are definitely null in the large mutual distances of the exterior problem.

This results into a structural difference between the interior and the exterior problem under which the probability of tunnel effects of free quarks can indeed be made identically null, e.g., by rendering incoherent the interior and exterior Hilbert spaces and other means.

Besides, the isotopic generalization of the SU(3) symmetry is locally isomorphic to the conventional symmetry (Mignani (1984)). As a result, the representation of the historical legagy of the ultimate nonlocality of the strong interactions via Santilli's isotopic techniques offers the possibility of genuine advances in quark theories, while leaving the unitary symmetries essentially unchanged.

The NOBEL COMMITTEE is warned against high ranking "false experts", and encouraged to dismiss superficial opinions expressed by individuals without an established record of expertise on the methods necessary for an ethically and scientifically sound judgment: the conditions of variational selfadjointness (see the encloseed Vol. I of Santilli's *Foundations of Theoretical Mechanics*).

I.5: INAPPLICABILITY OF CONVENTIONAL RELATIVITIES FOR THE INTERIOR DYNAMICAL PROBLEM.

The conventional relativities of contemporary physics are *inapplicable* (rather than "violated") for an effective characterization of interior dynamical systems (I.3) beyond any scientific doubt, for a variety of independent mathematical and physical reasons, such as:

1) The fundamental transformations of contemporary relativities, Galilei's, Lorentz's and Poincaré's transformations, are *linear* and *local*, as well known, while systems (I.3) are strictly *nonlinear* and *nonlocal*;

2) Contemporary relativities are centered on the *canonical-Hamiltonian* formalism, while systems (I.3) are strictly *nonhamiltonian* in the frame of their experimental detection;

3) Contemporary relativity are based on *Lie's symmetries* in their canonical realization, while the *conventional Lie's theory is fundamentally inapplicable for systems* (I.3);

4) Conventional relativities are centered on a *local-differential topology* (e.g., the Zeeman topology), while systems (I.3) require an essential *nonlocal-integral topology*;

5) Conventional relativities are centrally dependent on the *homogenuity and isotropy of space*, while interior physical media are manifestly *inhomogeneous and anisotropic*;

and numerous other independent technical reasons worked out by Santilli in all necessary details. In particular, the breakings of conventional symmetries for interior systems (I.3) were classified by Santilli (1978e), Sect. A.12, pp. 344-348) into: *isotopic, selfadjoint, semicanonical, canonical and essentially nonselfadjoint breakings.*

These studies establish beyond any credible scientific doubt the inapplicability for interior systems (I.3) of the mathematical foundations of Galilei's relativity, Einstein's special relativity and Einstein's general relativity, let alone the inapplicability of the relativities themselves, by therefore establishing the need to identify new mathematical methods and construct new coverinfg relativities.

As an illustration, ***the insistence in the exact validity for interior***

dynamical problems of the relativities for the exterior problem literally implies the acceptance of the perpetual motion in a physical environment, trivially, from the necessarily exact validity of their local rotational symmetry, with consequential necessary conservation of the angular momentum, without any possibility of escaping from these nonscientific conclusions because of the "No Reduction Theorems" recalled earlier.

SECTION II: CONCEPTUAL OVERVIEW OF SANTILLI'S DISCOVERIES

II.1: COMPREHENSIVE CHARACTER OF SANTILLI'S RESEARCH.

Santilli has conducted a truly vast study of the interior dynamical problem at the nonrelativistic, relativistic and gravitational levels, from a discrete and continuous viewpoint, as well as for classical and quantum mechanical treatments, all this repeated twice, one for the study of interior systems as closed-isolated (thus verifying conventional total conservation laws), and one for their open-nonconservative treatment (as conceived by Lagrange and Hamilton).

Moreover, in each of the above two main lines, Santilli constructed suitable generalizations of conventional mechanics, algebras and geometries, by resulting in this way in a novel scientific edifice of truly unique dimension, diversifications and interrelations, which is rather remarkable as the achievement by one single individual.

In a scientific edifice of this type, it appears recommendable to point out first the main conceptual lines, and then pass to a technical review.

II.2: SANTILLI'S CLOSED NONHAMILTONIAN SYSTEMS.

It is generally believed that the global stability of a system is due to the stability of the orbits of each constituent, as it is the case for the planetary and atomic structures.

Santilli (1978d) proved that, by no means, these systems exhaust all possible systems of the Universe. In fact, he identified a class of systems, at both classical and quantum mechanical levels, which he called *"closed nonhamiltonian" systems"*. These are systems whose total physical quantities are conventionally conserved (closure), but the internal orbits of the constituents are generally unstable because of contact interactions with the physical medium (nonhamiltonian character). In these broader systems we merely have *internal* exchanges of energy, angular momentum and other physical quantities, but in a way compatible with total conservations.

At the classical level, Santilli represented these novel systems with the equations

$$(\dot a^{\mu}) = \begin{pmatrix} \dot r_{ka} \\ \\ \dot p_{ka} \end{pmatrix} = \Gamma = (\Gamma^{\mu}(t, a, ..)) = \begin{pmatrix} p_{ka}/m_a \\ \\ -\dfrac{\partial V}{\partial r} + F_{ka}^{NSA}(t, r, p, ...) + \int_{\sigma} d\sigma \mathcal{F}_{ka}^{NSA}(t, r, p, ...) \end{pmatrix} \tag{II.1a}$$

$$\dot X_k = (\partial X_k / \partial a^{\mu})\, \dot a^{\mu} + \partial X_k / \partial t \equiv 0, \qquad k = 1, 2, ..., 10. \tag{II.1b}$$

where: the (ordered set of) ten conserved quantities X_k represent the conventional Galilean conservation laws of the energy, $\dot{H} = 0$, total linear momentum, $\dot{P} = 0$, total

JUPITER's STRUCTURE

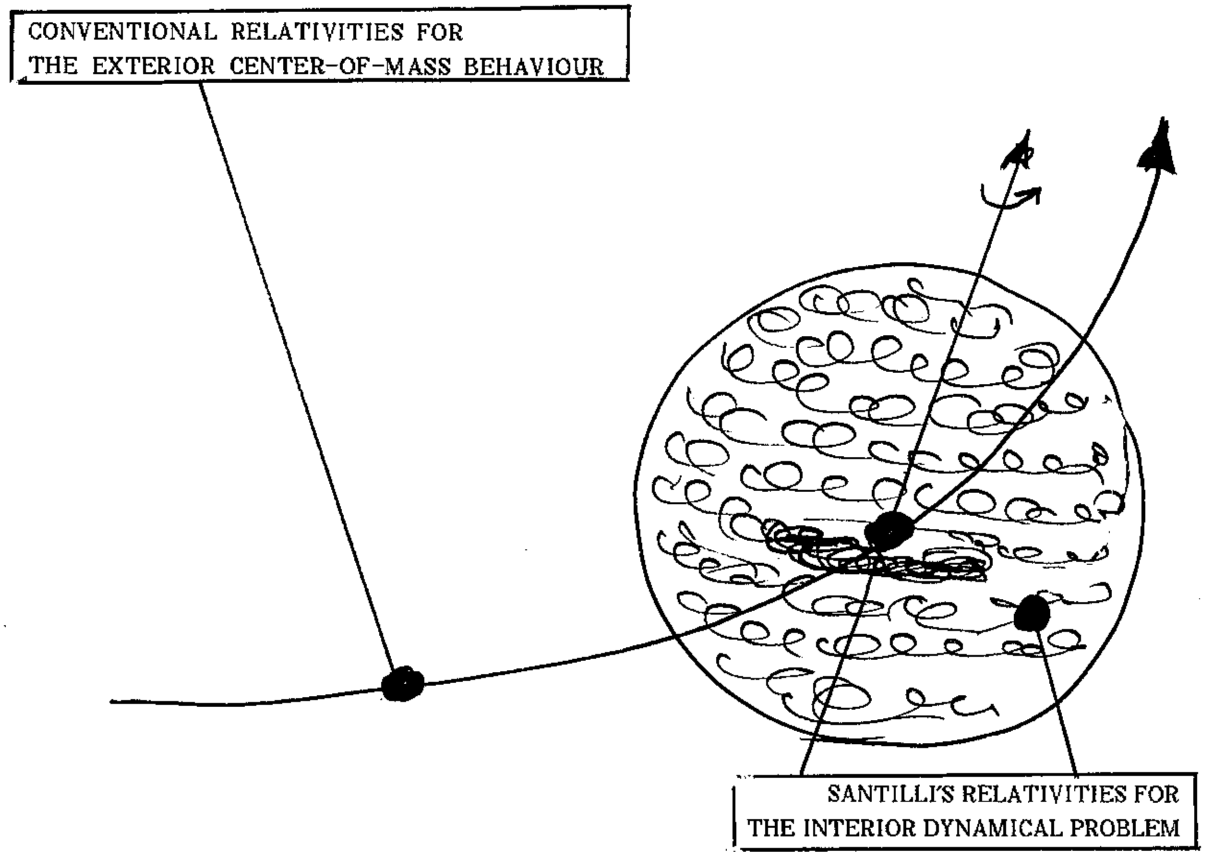

FIGURE II.1: A conceptual view often presented in Santilli's publications (1978c, d), (1981a), (1982a), etc.). The origin of all current relativities can be identified with the first visual observation of the Jovian system by Galileo Galilei back in 1609. Santilli's generalizations of Galilei's and Einstein's relativities can also be identified with a direct visual inspection of the Jovian systrem. The recent NASA missions to Jupiter clearly reveal a dichotomy of historical character: the exact *validity* of the conventional relativities for Jupiter's center-of-mass, EXTERIOR dynamics in the Solar system, jointly with the manifest *inapplicability* (and not "violation") of the same relativities for Jupiter's INTERIOR structural problem, as established by vortices with continuously varying angular momenta, etc. Also, when considered as isolated from the rest of the universe, Jupiter's is a majestic illustration of Santilli's "closed nonhamiltonian systems", because of its clear global stability and verification of conventional total conservation laws, while its interior dynamics is structurally nonconservative, nonlinear, nonlocal and nonhamiltonian. The above view has particularly conceptual value, because Santilli's generalized relativities provide a form-invariant description of Jupiter's structure conceived as aclosed nonhamiltonian system at the various Newtonian (Part I), relativistic (Part II) and gravitational (Part III) levels. As we shall see in Part IV, the operator counterpart of Jupiter's leads to a conceptually similar structure of hadrons, namely, an operator bound system whose center-of-mass

EXTERIOR dynamics (e.g., in a particle accelerator), verifies all conventional relativities, nevertheless, the INTERIOR structural problem is nonhamiltonian to acccount for the historical nonlocality of matter. This is the reason why Santilli repeats in his writings that the structure of hadrons may well be analytically equivalent to the structure of Jupiter.

angular momentum $\dot{J} = 0$, and uniform, motion of the center of mass, $\dot{G} = 0$, and NSA stands for variational nonselfadjointness, namely, for the violation of the integrability conditions for the existence of a Hamiltonian (see the enclosed Vol. I of Santilli's *Foundations of Theoretical Mechanics*).

Systems (II.1) constitute underdetermined systems of 6N differential equations with ten subsidiary constraints given by the total conservation laws, which are reducible for certain technical reasons to seven independent constraints. As such, the systems are consistent under sufficient continuity conditions generally verified in the physical reality.

In particular, Santilli proved that *unconstrained* solutions in the nonselfadjoint forces always exist for given potential forces.

In essence, Santilli's systems (II.1) establish that, in "closing" the satellite of Fig. I.1 with the surrounding atmosphere (or, equivalently, by "closing"Lagrange's and Hamilton's historical equations (I.1) via the inclusion of the medium in which motion occur), by no means, the contact, nonlocal, nonlagrangian or hamiltonian forces F_{ka} "disappear" according to a rather widespread but erroneous belief. On the contrary, Lagrange's and Hamilton's exterior forces F_{ka} persist in their entirety because variationally nonselfadjoint.

Closed nonhamiltonian systems for N = 2 were studied in detail in the original proposal (Santilli (1978d)). Their general theory was then studied in the enclosed Vol. II of Santilli's *Foundations of Theoretical Mechanics* (1982a). Additional basic advances were made in the memoir (Santilli (1988a)). Examples for N = 2 and 3 were worked out in details by Jannussis, Mijatovic and Veljanoski (1991) as the first examples of Santilli's generalization of Galilei's relativity (see Sect. V.7). Systems (II.1) were also studied from a statistical viewpoint by Fronteau et al. (1982), Tellez-arenas et al. (1982), and others.

Systems (II.1) are expected to have an intriguing connection with *Prigogine statistics* (see Prigogine (1962), (1968), (1990) and quoted references), which is currently under study by Jannussis et al. In fact, the systems recover the conventional time reversal symmetry for the exterior center-of-mass behavior, while admitting an intrinsically irreversible interior dynamics. The operator counterpart of the above results is presented in Part IV.

In summary, the central requirements for the closed-isolated approach to interior dynamical systems are the following:

I) All total, conventional, Galilean, Lorentzian or Riemannian conservations are verified.

II) The particles considered are extended-deformable while moving within a generally inhomogeneous and anisotropic medium; and

III) The forces are a combination of conventional local-potential, as well as nonlinerar, nonlocal and nonhamiltonian forces.

Note that, by conception and practical realization, no generalized interior dynamics can be detected from the outside, trivially, because of the exact validity of conventionbal relativities (Figure II.1).

This is nothing but a consequence of the fact that internal contact interactions for which the notion of potential energy has no meaning, cannot possibly have an impact on the exterior dynamical behaviour, as majestically established at the nonrelativistic level by Jupiter (Fig. II.1)

In Part II the NOBEL COMMITTEE will see the relativistic counterpart of the above setting. In fact, the historical legacy on the nonlocality of the hadronic structure can at best deal with *internal short range effects* also of nonpotential type[2] which, as such, cannot possibly affect the *exterior dynamical behaviour.* This leads to a dichotomy fully analogous of that of Jupiter, whereby the center-of-mass trajectory of a hadron in a particle accelerator strictly obeys Einstein's special relativity in a way fully compatible with the possible validity of Santilli's covering relativity for its interior structural problem.

As well known, *"closed Hamiltonian systems"* (i.e., isolated systems of point-like particles with only potential internal forces) constitute the physical foundations of contemporary relativities. Santilli's more general closed nonhamiltonian systems then constitute the physical foundations of his covering relativities. The main sections of this Nomination are therefore devoted to a review of the mathematical methods and relativities for closed nonhamiltonian systems

II.3: LAGRANGE'S AND HAMILTON'S OPEN SYSTEMS.

The second complementary approach identified by Santilli is the study of systems (I.3) as originally conceived by Lagrange's and Hamilton's, namely, with a total energy $H = T + V$ which is NONCONSERVED because of exchanges with the physical medium which is considered as external.

This alternative conception is a necesary complement of the preceding one on numerous counts indicated later on in this presentation, including the proper definition of ONE individual "particle" in interior conditions.

[2] A fundamental point for the consistency of this dichotomy is the *lack* of potential character of the contact internal forces, as conceived by Santilli and quantitatively treated via the *conditions of nonselfadjointness* . In fact, ***internal forces with a potential would directly affect the exterior dynamical behaviour, contrary to experimental evidence*** (Fig. II.1). This point is important for the NOBEL COMMITTEE to separate "true experts" from "false experts" (Sect.4). When exposed to interior nonlocal dynamical problems, the formers use the conditions of selfadjointness or other means to treat forces are they are in the physical reality, and proceeds to the consequential necessary *generalization* of current doctrines,. By contrast, the latters merely add "nonlocal potentials" to their trivial Lagranmgian or Hamiltonians, for the intent of *preserving* conventional relativities for the interior dynamics. This latter approach is however inconsistent on numerous mathematical and physical counts identified by Santilli (1981c) in details. In fact, the latter approach implies the invalidation of the Zeeman topology (evidently because strictly local-differential and therefore incompatible with "nonlocal potentials', with the consequential loss of the integrability to the finite Galilei's or Lorentz's symmetries. Physically, the addition of a "nonlocal potential" to a Lagrangian or a Hamiltonian implies the necessary alteration of the exterior trajectory (e.g., Jupiter's center-of-mass motion would be in part dependent on internal nonconservative effects), which is manifestly against clear experimental evidence.

Therefore, the contemporary study of the historical interior problem can be reinterpreted today as the study of *one particle in the most complex possible physical conditions* , e.g., a proton in the core of a star undergoing gravitational collapse. The complementarity of this open-nonconservative notion with that of the closed-isolated system as a whole is then evident, and equally evident is the need to identify the most effective means to treat each approach.

To clarify the above setting, recall that only one approach is needed for the characterization of a closed Hamiltonian system as a whole as well as for the characterization of one of its constituent. This is due to the fact that *both, the system as a whole and its individual constituents are in stable orbits.* A theory characterizing conservation laws is therefore sufficient for both profiles (see next section).

In the transition to closed nonhamiltonian systems the situation is, again, fundamentally different. In fact, in this case *the system as a whole is stable, thus requiring theories with the emphasis on conservation laws, while individual constituents are in generally unstable conditions, thus requiring more general theories which put the emphasis on time-rate-of-variations of physical quantities (see next section).*

This complementary open-nonconservative approach is outlined in Appendix A of this Nomination.

SECTION III: MATHEMATICAL OVERVIEW OF SANTILLI'S DISCOVERIES

III.1: STATEMENT OF THE PROBLEM.

As recalled in the foreword, the mathematical foundations of contemporary theoretical physics at large and, in particular, of the conventional Galilei's and Einstein's spacial relativities for the exterior problem, are given by *Lie algebras*, the *symplectic geometry*, and conventional *Hamiltonian mechanics*

The central equations for an exterior system of N particles in phase space are Hamilton's equations in their contemporary "truncated" form without external terms, which can be written in the local coordinates $a = (a^{\mu})$ of Eq.s (II.1)

$$\dot{a}^{\mu} = \omega^{\mu\nu} \frac{\partial H(a)}{\partial a^{\nu}}, \qquad H(a) = H(r,p) = T(p) + V(r), \ \mu = 1, 2, ..., 6N \qquad (III.1)$$

where the tensor $\omega^{\mu\nu}$, called the *canonical Lie tensor*, is given by

$$(\omega^{\mu\nu}) = \begin{pmatrix} 0_{3N\times3N} & I_{3N\times3N} \\ -I_{3N\times3N} & 0_{3N\times3N} \end{pmatrix} \qquad (III.2)$$

The underlying brackets are the familiar *Poisson brackets*

$$[A,B] = \frac{\partial A}{\partial a^{\mu}} \omega^{\mu\nu} \frac{\partial B}{\partial a^{\nu}} = \frac{\partial A}{\partial r_{ka}} \frac{\partial B}{\partial p_{ka}} - \frac{\partial B}{\partial r_{ka}} \frac{\partial A}{\partial p_{ka}}. \qquad (III.3)$$

$$k = 1, 2, 3, \qquad a = 1, 2, ..., N$$

Their most salient feature is that they characterize a Lie algebra, i.e.,. they verify the axioms

$$[A,B] + [B,A] = 0, \quad [A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0. \qquad (III.4)$$

from which the symplectic geometry and Hamiltonian mechanics follow, as well known (see, e.g., Abraham and Marsden (1967)).

But, the systems represented by these mathematical methods are local-differential and potential-Hamiltonian, while no effective mathematical method existed, at the time of initiation of Santilli's studies, for the treatment of interior, nonlinear, nonlocal and nonhamiltonian systems.

Thus, in order to be able to study of the generalization of current relativities, Santilli was forced to construct, first, suitable generalizations of their mathematical foundations, and then construct the covering relativities themselves.

Moreover, the novel mathematical methods had to be of dual character, a first class for the characterization of interior systems as closed-conservative (e.g., Jupiter's structure of Fig. II.1), and a second class for their open-nonconservative version (e.g., the satellite in Jupiter's atmosphere considered as external of Fig. I.1).

III.2: SANTILLI'S LIE-ISOTOPIC FORMULATIONS.

The formulations needed for the characterization of closed nonhamiltonian systems (II.1) must verify the following conditions:

1) Their algebra must possess a totally antisymmetric product, say $[A \hat{,} B] = -[B \hat{,} A]$ (or, equivalently, the underlying exterior calculus must be totally antisymmetric) as a necessary condition to represent the conservation of the total energy, $\dot{H} = [H \hat{,} H] \equiv 0$.

2) The formulations must be able to represent consistently nonlinear, nonlocal and nonhamiltonian forces, and must therefore have in particular a suitable nonlocal topology; and

3) All conventional formulations and disciplines must be contained as a particular case, and recovered identically when the nonhamiltonian forces are null.

The first identification of the above generalized methods was made by Santilli in his memoir (1978a) written at the *Lyman Laboratory of Physics* of *Harvard University* under contract from the U. S. Department of Energy No. ER-78-S-02-4742.A000 (see Docunment A), and then subsequently presented in the enclosed Vol. II of *Foundations of Theoretical Mechanics* (1982a). It consists of the discovery of a generalization of Lie's theory verifying conditions 1), 2) and 3) above, which he called *"Lie-isotopic theory"*, and which has been called in the literature the *Lie-Santilli theory* (see, .Kadeisvili (1991), Aringazin et al. (1992), and others).

In the subsequent years, he worked out the necessary details for the construction of the foundations of the various branches of his generalized theory, including the compatible formulation of fields, vector spaces, transformations, representations, analytic mechanics, symplectic and Riemannian geometries, ect. , as reviewed in the memoirs Santilli (1988a, b) and (1991a, b) (see the outline of Sect. IV below).

The central analytic equations are a generalization of the "truncated" Hamilton's equations (III.1) of the form, now called *Hamilton-Santilli equations*

$$\dot{a}^{\mu} = \omega^{\mu\alpha}\, \hat{1}_{\alpha}{}^{\nu}(a)\, \frac{\partial H(a)}{\partial a^{\nu}}, \quad H(a) = H(r,p) = T + V, \quad \mu, \nu, =1, 2, ..., N, \quad \text{(III.5)}$$

where $\hat{1}$ is the generalized unit of the theory (see next section), with generalized brackets